2.2.1 EXERCISES

To see all of the help resources associated with this section, click OSttS Chapter 2.

In Exercises 1 - 15, solve the equation.

For help with these exercises, click on one or more of the resources below:

- The definition of absolute value
- Solving equations involving the absolute value

1. $|x| = 6$
2. $|3x - 1| = 10$
3. $|4 - x| = 7$
4. $4 - |x| = 3$
5. $2|5x + 1| - 3 = 0$
6. $|7x - 1| + 2 = 0$
7. $\dfrac{5 - |x|}{2} = 1$
8. $\frac{2}{3}|5 - 2x| - \frac{1}{2} = 5$
9. $|x| = x + 3$
10. $|2x - 1| = x + 1$
11. $4 - |x| = 2x + 1$
12. $|x - 4| = x - 5$
13. $|x| = x^2$
14. $|x| = 12 - x^2$
15. $|x^2 - 1| = 3$

Prove that if $|f(x)| = |g(x)|$ then either $f(x) = g(x)$ or $f(x) = -g(x)$. Use that result to solve the equations in Exercises 16 - 21.

16. $|3x - 2| = |2x + 7|$
17. $|3x + 1| = |4x|$
18. $|1 - 2x| = |x + 1|$
19. $|4 - x| - |x + 2| = 0$
20. $|2 - 5x| = 5|x + 1|$
21. $3|x - 1| = 2|x + 1|$

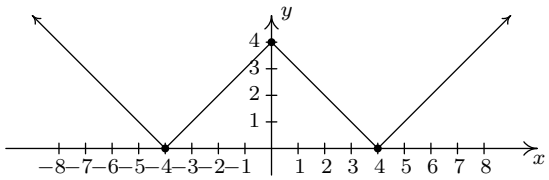
In Exercises 22 - 33, graph the function. Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

For help with these exercises, click on one or more of the resources below:

- Graphing absolute value functions
- Solving equations involving the absolute value
- The piecewise definition of absolute value
- Writing a function involving absolute values as a piecewise defined function

22. $f(x) = |x + 4|$
23. $f(x) = |x| + 4$
24. $f(x) = |4x|$

25. $f(x) = -3|x|$
26. $f(x) = 3|x+4| - 4$
27. $f(x) = \frac{1}{3}|2x-1|$
28. $f(x) = \frac{|x+4|}{x+4}$
29. $f(x) = \frac{|2-x|}{2-x}$
30. $f(x) = x + |x| - 3$
31. $f(x) = |x+2| - x$
32. $f(x) = |x+2| - |x|$
33. $f(x) = |x+4| + |x-2|$
34. With the help of your classmates, find an absolute value function whose graph is given below.



35. With help from your classmates, prove the second, third and fifth parts of Theorem 2.1.
36. Prove **The Triangle Inequality:** For all real numbers a and b, $|a+b| \leq |a| + |b|$.

Checkpoint Quiz 2.2

1. Let $f(x) = 2|x| - |x-3|$.

   (a) Use the definition of absolute value to rewrite f as a piecewise-defined function.

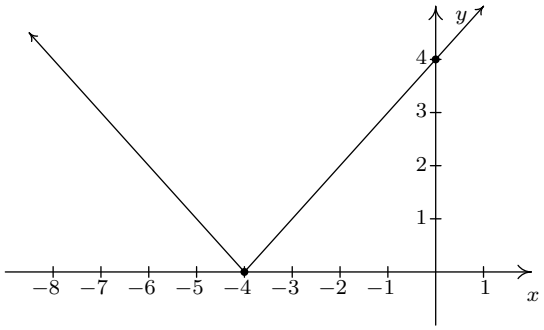
   (b) Graph $y = f(x)$. Find the zeros of f and the x- and y-intercepts of the graph. From the graph, determine the domain and range of f, list the intervals on which f is increasing, decreasing, or constant, and find the relative and absolute extrema, if they exist.

For worked out solutions to this quiz, click the links below:

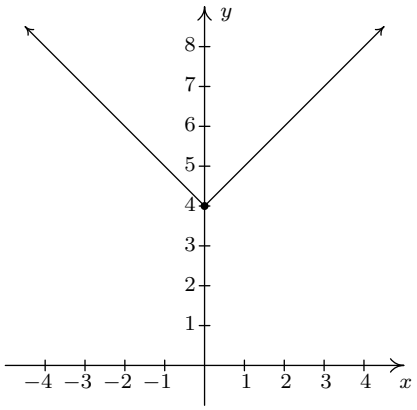
- Quiz Solution Part 1
- Quiz Solution Part 2

2.2.2 ANSWERS

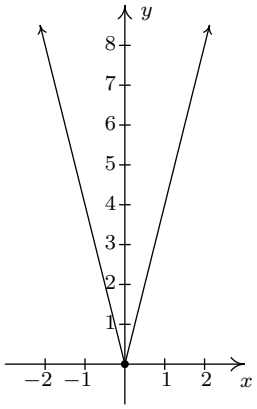
1. $x = -6$ or $x = 6$
2. $x = -3$ or $x = \frac{11}{3}$
3. $x = -3$ or $x = 11$
4. $x = -1$ or $x = 1$
5. $x = -\frac{1}{2}$ or $x = \frac{1}{10}$
6. no solution
7. $x = -3$ or $x = 3$
8. $x = -\frac{13}{8}$ or $x = \frac{53}{8}$
9. $x = -\frac{3}{2}$
10. $x = 0$ or $x = 2$
11. $x = 1$
12. no solution
13. $x = -1$, $x = 0$ or $x = 1$
14. $x = -3$ or $x = 3$
15. $x = -2$ or $x = 2$
16. $x = -1$ or $x = 9$
17. $x = -\frac{1}{7}$ or $x = 1$
18. $x = 0$ or $x = 2$
19. $x = 1$
20. $x = -\frac{3}{10}$
21. $x = \frac{1}{5}$ or $x = 5$
22. $f(x) = |x + 4|$
    $f(-4) = 0$
    x-intercept $(-4, 0)$
    y-intercept $(0, 4)$
    Domain $(-\infty, \infty)$
    Range $[0, \infty)$
    Decreasing on $(-\infty, -4]$
    Increasing on $[-4, \infty)$
    Relative and absolute min. at $(-4, 0)$
    No relative or absolute maximum



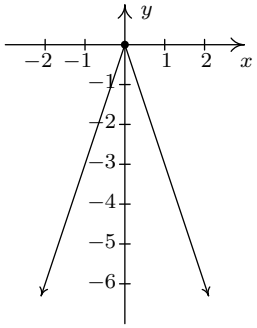
23. $f(x) = |x| + 4$
    No zeros
    No x-intercepts
    y-intercept $(0, 4)$
    Domain $(-\infty, \infty)$
    Range $[4, \infty)$
    Decreasing on $(-\infty, 0]$
    Increasing on $[0, \infty)$
    Relative and absolute minimum at $(0, 4)$
    No relative or absolute maximum



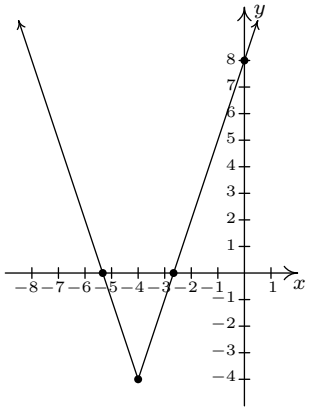
24. $f(x) = |4x|$
$f(0) = 0$
x-intercept $(0, 0)$
y-intercept $(0, 0)$
Domain $(-\infty, \infty)$
Range $[0, \infty)$
Decreasing on $(-\infty, 0]$
Increasing on $[0, \infty)$
Relative and absolute minimum at $(0, 0)$
No relative or absolute maximum



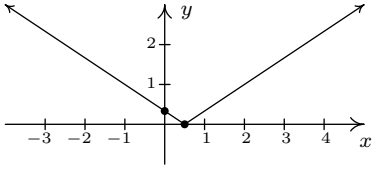
25. $f(x) = -3|x|$
$f(0) = 0$
x-intercept $(0, 0)$
y-intercept $(0, 0)$
Domain $(-\infty, \infty)$
Range $(-\infty, 0]$
Increasing on $(-\infty, 0]$
Decreasing on $[0, \infty)$
Relative and absolute maximum at $(0, 0)$
No relative or absolute minimum



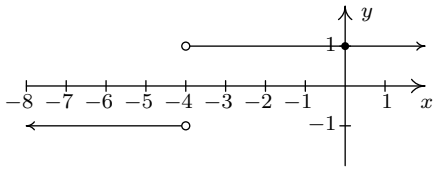
26. $f(x) = 3|x + 4| - 4$
$f\left(-\frac{16}{3}\right) = 0$, $f\left(-\frac{8}{3}\right) = 0$
x-intercepts $\left(-\frac{16}{3}, 0\right)$, $\left(-\frac{8}{3}, 0\right)$
y-intercept $(0, 8)$
Domain $(-\infty, \infty)$
Range $[-4, \infty)$
Decreasing on $(-\infty, -4]$
Increasing on $[-4, \infty)$
Relative and absolute min. at $(-4, -4)$
No relative or absolute maximum



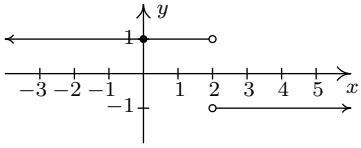
27. $f(x) = \frac{1}{3}|2x - 1|$
$f\left(\frac{1}{2}\right) = 0$
x-intercepts $\left(\frac{1}{2}, 0\right)$
y-intercept $\left(0, \frac{1}{3}\right)$
Domain $(-\infty, \infty)$
Range $[0, \infty)$
Decreasing on $\left(-\infty, \frac{1}{2}\right]$
Increasing on $\left[\frac{1}{2}, \infty\right)$
Relative and absolute min. at $\left(\frac{1}{2}, 0\right)$
No relative or absolute maximum



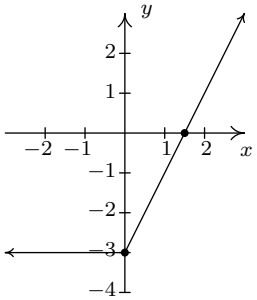
28. $f(x) = \dfrac{|x+4|}{x+4}$
No zeros
No x-intercept
y-intercept $(0, 1)$
Domain $(-\infty, -4) \cup (-4, \infty)$
Range $\{-1, 1\}$
Constant on $(-\infty, -4)$
Constant on $(-4, \infty)$
Absolute minimum at every point $(x, -1)$ where $x < -4$
Absolute maximum at every point $(x, 1)$ where $x > -4$
Relative maximum AND minimum at every point on the graph



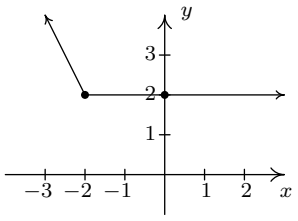
29. $f(x) = \dfrac{|2-x|}{2-x}$
No zeros
No x-intercept
y-intercept $(0, 1)$
Domain $(-\infty, 2) \cup (2, \infty)$
Range $\{-1, 1\}$
Constant on $(-\infty, 2)$
Constant on $(2, \infty)$
Absolute minimum at every point $(x, -1)$ where $x > 2$
Absolute maximum at every point $(x, 1)$ where $x < 2$
Relative maximum AND minimum at every point on the graph



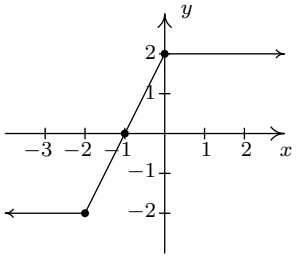
30. Re-write $f(x) = x + |x| - 3$ as
$$f(x) = \begin{cases} -3 & \text{if} \quad x < 0 \\ 2x - 3 & \text{if} \quad x \geq 0 \end{cases}$$
$f\left(\frac{3}{2}\right) = 0$
x-intercept $\left(\frac{3}{2}, 0\right)$
y-intercept $(0, -3)$
Domain $(-\infty, \infty)$
Range $[-3, \infty)$
Increasing on $[0, \infty)$
Constant on $(-\infty, 0]$
Absolute minimum at every point $(x, -3)$ where $x \leq 0$
No absolute maximum
Relative minimum at every point $(x, -3)$ where $x \leq 0$
Relative maximum at every point $(x, -3)$ where $x < 0$



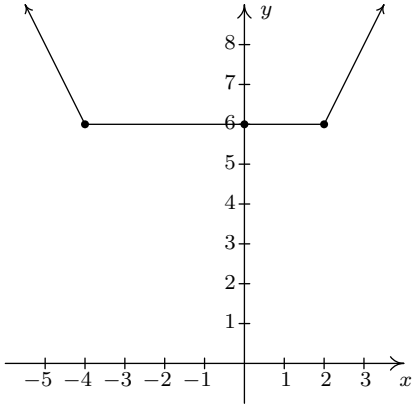
31. Re-write $f(x) = |x+2| - x$ as
$$f(x) = \begin{cases} -2x-2 & \text{if} \quad x < -2 \\ 2 & \text{if} \quad x \geq -2 \end{cases}$$
No zeros
No x-intercepts
y-intercept $(0, 2)$
Domain $(-\infty, \infty)$
Range $[2, \infty)$
Decreasing on $(-\infty, -2]$
Constant on $[-2, \infty)$
Absolute minimum at every point $(x, 2)$ where $x \geq -2$
No absolute maximum
Relative minimum at every point $(x, 2)$ where $x \geq -2$
Relative maximum at every point $(x, 2)$ where $x > -2$



32. Re-write $f(x) = |x+2| - |x|$ as
$$f(x) = \begin{cases} -2 & \text{if} \quad x < -2 \\ 2x+2 & \text{if} \quad -2 \leq x < 0 \\ 2 & \text{if} \quad x \geq 0 \end{cases}$$
$f(-1) = 0$
x-intercept $(-1, 0)$
y-intercept $(0, 2)$
Domain $(-\infty, \infty)$
Range $[-2, 2]$
Increasing on $[-2, 0]$
Constant on $(-\infty, -2]$
Constant on $[0, \infty)$
Absolute minimum at every point $(x, -2)$ where $x \leq -2$
Absolute maximum at every point $(x, 2)$ where $x \geq 0$
Relative minimum at every point $(x, -2)$ where $x \leq -2$ and at every point $(x, 2)$ where $x > 0$
Relative maximum at every point $(x, -2)$ where $x < -2$ and at every point $(x, 2)$ where $x \geq 0$



33. Re-write $f(x) = |x+4| + |x-2|$ as
$$f(x) = \begin{cases} -2x-2 & \text{if} \quad x < -4 \\ 6 & \text{if} \quad -4 \leq x < 2 \\ 2x+2 & \text{if} \quad x \geq 2 \end{cases}$$
No zeros
No x-intercept
y-intercept $(0, 6)$
Domain $(-\infty, \infty)$
Range $[6, \infty)$
Decreasing on $(-\infty, -4]$
Constant on $[-4, 2]$
Increasing on $[2, \infty)$
Absolute minimum at every point $(x, 6)$ where $-4 \leq x \leq 2$
No absolute maximum
Relative minimum at every point $(x, 6)$ where $-4 \leq x \leq 2$
Relative maximum at every point $(x, 6)$ where $-4 < x < 2$



35. $f(x) = ||x| - 4|$